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DISPLACEMENT OF BODIES WITH TOSSING ON A VIBRATING SURFACE

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A new approach to investigations of structural mechanisms that accomplish displacement of bodies with tossing on a vibrating surface is suggested that explains some facts that are widely known from experiments. This approach makes it possible to select criteria for optimization of the geometric parameters and physical charactersitics of the objects created that provide a reduction in mechanism loading and economy in energy resources.

Mathematical simulation and analysis of the kinematics and dynamics of vibration-transporting devices that provide directional displacement of loads with tossing are of obvious interest despite existing experience in designing these mechanisms and a large amount of experimental materials, filmings, and photographs. A review of theoretical investigations known at the present stage is given in [1-6].

Figure 1 presents a diagram of a vibration-transporting device in the form of a vibrating tray that reciprocates. Therefore the velocities and accelerations of all points of the tray DP will be equal in magnitude. Their analytical dependences can be expressed by the formulas

$$v_{Px} = -r\omega \frac{\sin(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} \cos\varphi_1 , \quad (1)$$

$$v_{Py} = r\omega \frac{\sin(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} \sin\varphi_1 , \quad (2)$$

$$a_{Px} = -\omega^2 r \cos\varphi_1 \frac{\cos(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} - \frac{\omega^2 r^2}{l_3} \frac{\cos^2(\varphi + \varphi_1)}{\cos^3(\varphi_1 - \theta_1)} \cos\varphi_1 + \frac{\omega^2 r^2}{l_4} \frac{\sin^2(\varphi + \theta_1)}{\cos^3(\varphi_1 - \theta_1)} \sin\theta_1 , \quad (3)$$

$$a_{Py} = \omega^2 r \sin\varphi_1 \frac{\cos(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} + \frac{\omega^2 r^2}{l_3} \frac{\cos^2(\varphi + \varphi_1)}{\cos^3(\varphi_1 - \theta_1)} \sin\varphi_1 + \frac{\omega^2 r^2}{l_4} \frac{\sin^2(\varphi + \theta_1)}{\cos^3(\varphi_1 - \theta_1)} \cos\theta_1 , \quad (4)$$

Here $\varphi = \varphi(t)$, $\theta_1 = \theta_1(t)$, and $\varphi_1 = \varphi_1(t)$ are known functions of time t prescribed numerically. And the kinematic parameters of all the points of the mechanism are uniquely determined. Of greatest interest is the position of the point P, from which the transportation (displacement) of material bodies over the tray occurs. In a Cartesian system of coordinates xOy, related to the crankgear, the position of the point P can be represented in the following form:

$$x_P = x_E - l_4 \sin\varphi_1 + l_{16} \cos\beta , \quad (5)$$

$$y_P = y_E - l_4 \cos\varphi_1 - l_{16} \sin\beta , \quad (6)$$

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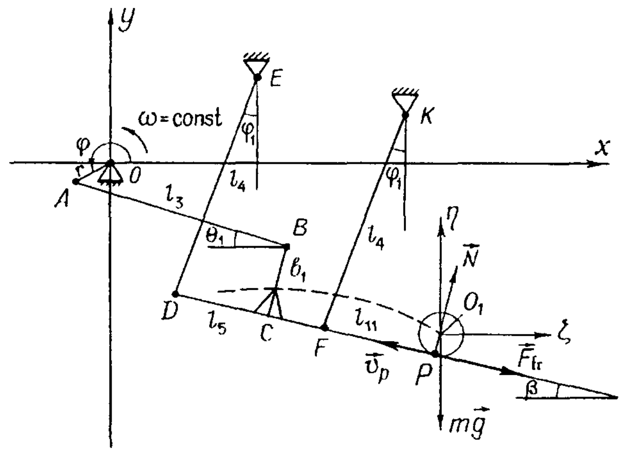

Fig. 1. Diagram of a screen-type mechanism.

where x_E and y_E are fixed values of the coordinates of the fixed point E; $l_{16} = l_5 + l_{11}$; l_4, l_5, l_{11} are the lengths of the mechanism units (Fig. 1); $\beta = \text{const}$.

The differential equation of motion of a body along the tray has the form

$$m\,(\ddot{x}_P + \ddot{\xi}_P) = N\,(t)\sin\beta + F_{\text{fr}}\,(t)\cos\beta\,, \tag{7}$$

$$m\,(\ddot{y}_P + \ddot{\eta}_P) = -\,mg + N\,(t)\cos\beta - F_{\text{fr}}\,(t)\sin\beta\,, \tag{8}$$

where $\eta O_1 \xi$ is a system of coordinates associated with the tray surface; $\ddot{x}_P(t) = a_{P_x}(t)$, $\ddot{y}_P(t) = a_{P_y}(t)$ are known analytical functions established on the basis of the mechanism kinematics from formulas (3) and (4). By Amonton's hypothesis [4], the dependence of the dry-friction force without slipping between the tray and the body is as follows:

$$\vec{F}_{\text{fr}}\,(t) = fN\,(t)\,\text{sign}\,\vec{v}(t)\,, \tag{9}$$

where $\vec{v}(t)$ is the relative velocity of the body over the tray with the components $(\dot{\xi}(t);\ \dot{\eta}(t))$; $\text{sign}\,\vec{v}(t) = \vec{i}\,\text{sign}\,\dot{\xi}(t) + \vec{j}\,\text{sign}\,\dot{\eta}(t)$; $\vec{i}$ and $\vec{j}$ are the unit vectors along the axes ξ and η.

Separation of bodies lying on the tray is possible during its lift in which $v_{P_x}(t) < 0$ and $v_{P_y}(t) > 0$. The equations of motion of a body in the relative system of coordinates take the form

$$\ddot{\xi}_P = -\,\ddot{x}_P + \frac{1}{m}\,N\,(t)\,(\sin\beta + f\cos\beta) \equiv \frac{1}{m}\,R_\xi\,(t)\,, \tag{10}$$

$$\ddot{\eta}_P = -\,\ddot{y}_P - g + \frac{1}{m}\,N\,(t)\,(\cos\beta - f\sin\beta) \equiv \frac{1}{m}\,R_\eta\,(t)\,. \tag{11}$$

At the moment of separation of the body from the tray the normal-pressure force $N(t) \le 0$. Then, from formulas (10) and (11) at the moment of body flight $N(t) = 0$

$$\ddot{\xi}_P = -\,\ddot{x}_P\,, \tag{12}$$

$$\ddot{\eta}_P = -\,\ddot{y}_P - g\,. \tag{13}$$

Or, which is the same,

$$\frac{1}{m}\,R_\xi\,(t) = -\,\ddot{x}_P\,, \tag{14}$$

$$\frac{1}{m} R_\eta (t) = - \ddot{y}_P - g . \tag{15}$$

System of equations (14) and (15) is incomplete, since $R_\zeta(t)$, $R_\eta(t)$, and the instant of time t are unknown quantities. The missing third condition is found from physical considerations: separation of the body from the tray surface is possible at the instant of time $t = t_\Delta$, when the resultant force $\vec{R}$ with the components $(R_\zeta(t), R_\eta(t))$ becomes directional to the tray surface DP (Fig. 1) corresponding to the beam $O_1 W$ on the hodograph of accelerations, since in any another position the body will be pressed by this force to its surface or will slide over it. Then

$$R_\eta (t) = - \tan \beta \, R_\zeta (t) \, , \quad \beta = \text{const} . \tag{16}$$

We can easily implement this algorithm numerically, but the derivation of simple analytical relations is of greatest interest. To do this, we represent the hodograph of the acceleration a_{P_y} by the following square-law dependence:

$$a_{Py} = a_0 + a_1 \, a_{Px} + a_2 \, a_{Px}^2 , \tag{17}$$

where

$$a_0 = \frac{r^2 \omega^2}{l_4} \frac{1}{\cos \varphi_1 \cos^2 (\varphi_1 - \theta_1)} + \frac{r^3 \omega^2}{l_3 l_4} \frac{\cos^2 (\varphi + \varphi_1)}{\cos \varphi_1 \cos^4 (\varphi_1 - \theta_1)} \times$$

$$\times \left(2 \cos (\varphi + \theta_1) + \frac{r}{l_3} \frac{\cos^2 (\varphi + \varphi_1)}{\cos^2 (\varphi_1 - \theta_1)} \right) , \tag{18}$$

$$a_1 = - \tan \varphi_1 , \tag{19}$$

$$a_2 = - \frac{1}{l_4 \omega^2 \cos^3 \varphi_1} . \tag{20}$$

According to formulas (14), (15), (16), and (17), the intersection of the hodograph and the straight line $a_{P_y} = -\tan \beta a_{P_x}$ leads to the relation

$$a_{Px} = \frac{\tan \varphi_1 - \tan \beta}{2 \, |a_2|} - \sqrt{\left(\left(\frac{\tan \varphi_1 - \tan \beta}{2 a_2} \right)^2 + \frac{a_0 + g}{|a_2|} \right)} . \tag{21}$$

Then, to determine $\varphi(t_\Delta)$ at the moment of separation, from formulas (3) and (21) we obtain a transcendental equation in which $\theta_1 = \theta_1 (\varphi(t_\Delta))$ and $\varphi_1 = \varphi_1 (\varphi(t_\Delta))$:

$$- \omega^2 r \cos \varphi_1 \frac{\cos (\varphi + \theta_1)}{\cos (\varphi_1 - \theta_1)} - \frac{\omega^2 r^2}{l_3} \frac{\cos^2 (\varphi + \varphi_1)}{\cos^3 (\varphi_1 - \theta_1)} \cos \varphi_1 + \frac{\omega^2 r^2}{l_4} \frac{\sin^2 (\varphi + \theta_1)}{\cos^3 (\varphi_1 - \theta_1)} \sin \theta_1 =$$

$$= - A \left(1 - \sqrt{B - |x|}\right) , \tag{22}$$

where

$$A = \frac{l_4 \omega^2}{2 \cos \beta} \cos^2 \varphi_1 \sin (\varphi_1 - \beta) ;$$

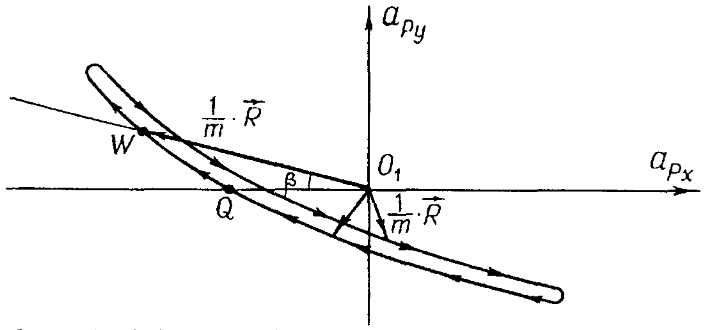

Fig. 2. Hodograph of the inertial force of a material body lying on a vibrating surface, referred to unit mass.

$$B = 1 + \frac{4g\cos^2\beta}{l_4\omega^2\cos\varphi_1\sin^2(\varphi_1-\beta)} + \frac{4r^2\cos^2\beta}{l_4^2\cos^2\varphi_1\sin^2(\varphi_1-\beta)\cos^2(\varphi_1-\theta_1)};$$

$$x = \frac{8r^3\cos^2\beta\cos(\varphi+\theta_1)\cos^2(\varphi+\varphi_1)}{l_3 l_4^2\cos^2\varphi_1\sin^2(\varphi_1-\beta)\cos^4(\varphi_1-\theta_1)}.$$

Neglecting small quantities that have the factors $r/l_3 \ll 1$ and $r/l_4 \ll 1$ in formula (22) and having chosen, as φ_1 and θ_1, the average values from the limits of their changes, we obtain an approximate formula for finding the angle $\varphi(t_\Delta)$ at the moment of separation:

$$\cos(\varphi+\theta_1) = -\frac{g}{\omega^2 r}\frac{\cos\beta\cos(\varphi_1-\theta_1)}{\sin(\varphi_1-\beta)}, \qquad (23)$$

where

$$\varphi = -\theta_1 + \pi - \arccos\left(\frac{g}{\omega^2 r}\frac{\cos\beta\cos(\varphi_1-\theta_1)}{\sin(\varphi_1-\beta)}\right). \qquad (24)$$

In calculating the angle $\varphi(t_\Delta)$ by formula (24), the error amounts to 3–7%, and therefore to obtain accurate results, it is convenient to carry out numerical calculations using nonlinear transcendental equation (22).

On the basis of the aforesaid it can be concluded that the suggested procedure (14)-(17) is a fundamentally new method of calculating the kinematic characteristics of separation and flight of particles and material bodies. This approach gives an explanation of certain established but until the present time unjustified facts that are widely presented in the scientific literature [3-6]. One of these facts is the delay of the tossing of a soil layer with potatoes on the screen of a potato harvester [3-5]. This behavior of the soil layer was explained by the elasticity of its strata. The fact itself cannot be denied; however, in our opinion, the reason for the delay is that research workers took the condition $a_{P_y} = -g$ as the moment t_* of separation of the material bodies. This corresponds to the point Q on the curve of the hodograph of accelerations (Fig. 2), but actually the moment $t_\Delta = t_* + \Delta t$ corresponds to the point W spaced from Q by Δt = 0.001–0.01 sec and $\Delta\varphi$ = 5–25 deg, calculated for the specific parameters of the mechanism that were involved in our experiments. In view of this, the height and the distance of flight of material bodies change, since the initial conditions at the moment of separation, corresponding to the point W on the hodograph of accelerations, are different (Fig. 2). Their magnitudes are smaller in comparison with those of the velocities at the point Q. Therefore the height of the tossing determined by the procedure suggested is decreased on the average by a factor of 1.5 compared to the procedure known in the literature [3-5] and corresponds ideally to the experimental data obtained by us and results presented in monographs of a number of authors.

NOTATION

r, φ, ω, radius, angle of rotation, and angular velocity of the crank; t, time coordinate; l_i, $i = \overline{1, 16}$, lengths of crankgear units (Fig. 1); $\theta_1(t)$ and $\varphi_1(t)$, angles of rotation of the units l_3 and l_4; x_E, y_E, x_P, y_P, coordinates of the points E and P; m, mass of the material object; g, acceleration of free fall; β, dip angle of the tray surface DP to the horizonal; $N(t)$, normal reaction of the tray to the body; $F_{fr}(t)$, force of friction between the tray and the body; f, coefficiet of friction; $v_{P_x}(t)$ and $v_{P_y}(t)$, velocities of the material object located at the point P; $\vec{R} = (R_\zeta(t), R_\eta(t))$, total force acting on the body.

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